Image Comparisons of Black Hole vs. Neutron Dark Star by Ray Tracing

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ABSTRACT

In previous papers we have discussed the concept of a theory of gravitation with local energy conservation, and the properties of a large neutron star resulting when the energy of gravitation resides locally with the particle mass and not in the gravitational field. A large neutron star's surface radius grows closer to the gravitational radius as the mass increases. **Since the localization of energy applies to the photon, a photon does not decrease in energy rising in a gravitational field, and can thus escape.** Photon trajectories in a strong gravitational field can be investigated by the use of ray tracing procedures. Only a fraction of the blackbody radiation emitted from the surface escapes into space (about 0.00004% for Sag A*). Because of the low % of escaping radiation, the heavy neutron stars considered in this paper will be referred to as a Neutron Dark Star (NDS). In contrast to the Black Hole (BH) which should be totally dark inside the photon shadow, the NDS will appear as a fuzzy low luminosity ball. For Sag A* a Full Width Half Maximum diameter is about 3.85 Schwarzschild radii inside the shadow. (http://www.arxdtf.org/css/Image%20Comparisons.pdf).

The Event Horizon Telescope capable of resolving the Black Hole Sagittarius A* is under construction and is currently in the process of taking measurements. The merits of this paper are going to be evaluated very shortly.

Introduction

Early on in the development of GR Hilbert recognized that the theory had an ”improper energy theorem” that is, one could define a divergence free quantity, analogous to the momentum density of Special Relativity, but it is quite arbitrary and is gauge dependent. It is not covariant under a general coordinate transformation, or more simply there is no local conservation of energy. In a defined volume of space the change of energy inside, is not the sum of the energy entering and leaving through the surface.

Emmy Noether formalized the issue 1918 in a definitive paper "Invariante Varlations Probleme" illustrating the problem. Noether’s theorem definitively shows that contrary to all other forces, energy cannot be conserved nor localized in a Riemannian gauge field representation. **It is presumed here that this is a flaw in GR, and it is asserted here that Noether’s theorem is not an indicator of a physical reality, but an indicator of the approximate nature of GR**. This can best be tested in the observation of the properties of objects cited as being black holes.

From “Scalar Gravitational Theory with Variable Rest Mass” [1] the local conservation of energy is satisfied when:

$$M^2\left(1-\frac{v^2}{c^2}\right)=M_0^2\left(1-\frac{\mu}{r}\right)^2 \quad , \tag{1.1}$$

This is postulated defining relationship between velocity, mass, and gravitation, for the local conservation of energy. It assumes the status of that for the Ricci Tensor in General Relativity.

The local rest mass can be defined as:

$$m_{Local} = m_0^2\left(1-\frac{\mu}{r}\right)^2 \qquad r\to\mu \quad , \quad m_{Local}\to 0$$

Dividing by the right bracket gives the standard but approximate relativistic Lagrangian.

$$L = M_0c^2 = \left(Mc^2 - \frac{1}{2}Mv^2 + \frac{GMm}{r}\right) \tag{1.3}$$

In a conservative system a particle infalling from infinity

$$m = m_0$$

and

$$\left(1-\frac{v^2}{c^2}\right)=\left(1-\frac{\mu}{r}\right)^2 \qquad (1.4)$$

As the particle velocity goes to c the rest mass goes to zero.

In a locally conserved conservative system with the test mass a function of the radius it is easily shown that the velocity of light is :

$$M^2\left(1-\frac{v^2}{c^2}\right)=M_0^2\left(1-\frac{\mu}{r}\right)^2$$
$$\downarrow \qquad (1.5)$$
$$\frac{c}{c_0}=\left(1-\frac{\mu}{r}\right)^2$$

Which is only slightly different value in flat space of the Shapiro velocity projected in three space with constant t by (Blandford & Thorne) of:

$$\frac{c}{c_0}=\left(1-\frac{2\mu}{r}\right) \quad , \qquad (1.6)$$

Ray Traced Method

The comparison of the appearance of a Neutron Dark Star (NDS) to a Black Hole (BH) will be made by the use of ray tracing techniques of the photon trajectories. For GR, the index of refraction for photons traversing flat space can be found using the general form of a static and spherically symmetric metric [4][5],

$$ds^2 - B(r)dt - A(r)dr^2 - r^2\left(d\theta^2 + \mathbf{sin}^2\theta d\phi\right)$$
$$A = \left(1 - \frac{2\mu}{r}\right)^{-1} \quad B = \frac{2\mu}{r^2}\left(1 - \frac{2\mu}{r}\right)^{-1} \tag{1.8}$$

This has an index of refraction (c/c_0) value in flat space (Blandford & Thorne) of:

$$\frac{c}{c_0} = \left(1 - \frac{2\mu}{r}\right) \quad , \tag{1.9}$$

or the more detailed analytic expression of Karimi & Khorasani [7]:

$$\frac{c}{c_0} = \left\{1 - \frac{1}{2}\left[\frac{r}{r_s} - \frac{1}{2} + \sqrt{\left(\frac{r}{r_s}\right)^2 - \frac{r}{r_s}}\right]^{-1}\right\}$$
$$* \left\{1 + \frac{1}{2}\left[\frac{r}{r_s} - \frac{1}{2} + \sqrt{\left(\frac{r}{r_s}\right)^2 - \frac{r}{r_s}}\right]^{-1}\right\}^{-3} \tag{1.10}$$

So the comparison is essentially between:

$$\frac{c}{c_0} = \left(1 - \frac{\mu}{r}\right)^2 \quad \text{AND} \quad \frac{c}{c_0} = \left(1 - \frac{2\mu}{r}\right)$$

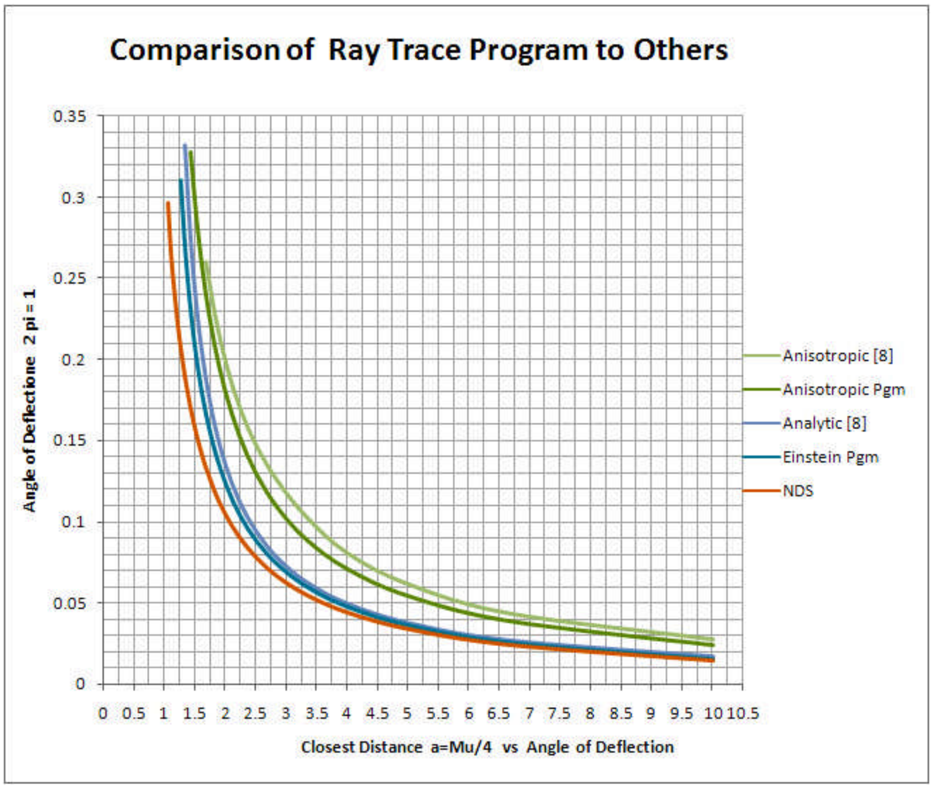
Ray Trace Algorithm

The algorithm used in this paper is a point to point implementation of Fermats principle using Snell's law and the index of refraction. The routine is constructed in quick basic and has been tested against other ray trace routines, and analytical solutions. The difference has been found to be 1-2 percent in the deflection angles of particles passing near black holes from a range of 4 to 80μ (gravitational radii). The results for the NDS could be slightly in error as a result of the inherent errors in the algorithm, but the error must be small, and the substantial difference between the NDS and the GR Black Hole theory results are qualitatively unambiguous.

A comparison of the algorithm output shown in Figure A1, to those of Karimi, Khorasani using Mathematica [7],[8], demonstrates near equivalent results.

Quick Basic Double Precision Ray Trace Program Excerpt

```
                        Gravitational Ray Trace
                        '             DT Froedge
                        '           copyright 2014
                        '   GR black hole, and VRM Dark Star


                 'This is a basic program that calculates ray traces
                 'in the vicinity of black holes and variable mass stars
                 'calculations are all in double precision.
                 'Output is a comma delimited text file that can be imported into
                 Excel
                 'or other spreadsheets for graphing.
                 'Program calculates in the first quadrant (quad)by rotation
                 'then corrects output to proper quadrant by reversing this.
                 'program only handles counter clockwise motion.
                 'distance is in gravitational radii. c=1, Mu = 1
                 'There are 10000 iterations between print iterations

fprmat% = 2           'pick here User choice of 1-5 index of refraction formulas

                       pi# = 3.14159265358979#
                       x1# = 6               'initial x position in Mu  (to be set by
user)
                       y1# = 0               'initial y position in Mu  (to be set by
user)
                       P1# = pi# / 2         'initial angle in radians  (to be set by
user)

CLS
'Sag a*        mass 4.1 e6   0.6 e6 suns          =  8.154572e39  grams
'Gm/c^2=     Mu = 605181439025  cm     =  20.186  light seconds


quad% = 1                         'start quadrant
Delt# = .00001                    'increments of time
sht% = 0                          'increment counter
sht2% = 0                         'incremental outputs data after Sht% counts out
r0# = (xo# ^ 2 + yo# ^ 2) ^ .5    'initial radius       defined
a#(2) = 0
st% = 2

xxo# = x1#
yyo# = y1#
po# = P1#
fun% = -1

a#(2) = 0
a#(1) = 0
'                                 'Some Index of refraction formulas tested
IF fprmat% = 1 THEN Formprt$ = " NDS     v = (1 - 1 / r#) ^ 2 "
IF fprmat% = 2 THEN Formprt$ = "Einstein v = (1 - 2 / r#) "
IF fprmat% = 3 THEN Formprt$ = "Karimi Isotropic   Eq 5b"
IF fprmat% = 4 THEN Formprt$ = "Karimi Anisotropi  Eq 13"
IF fprmat% = 5 THEN Formprt$ = "Karimi Component   Eq.14"
```



Blue are comparison with GR analytic Solution
Green are comparison with Karimi algorithm .
NDS Dark Star Angle of deflection .

Overall Parameters

The mass of Sag A* is about 4.31 ± 0.38 e6 solar masses and if is a Black Hole will have a shadow radius of about 2.6 r_S . From calculations in [2], a **NDS the size of Sag A* has a solid surface about 1.025 times the gravitational radius.**

Ray Tracing

From T. Lacroix & J. Silk,[4] a semi analytical derivation of the Shadow of a GR Black Hole from the field equations is illustrated in Figure 1.

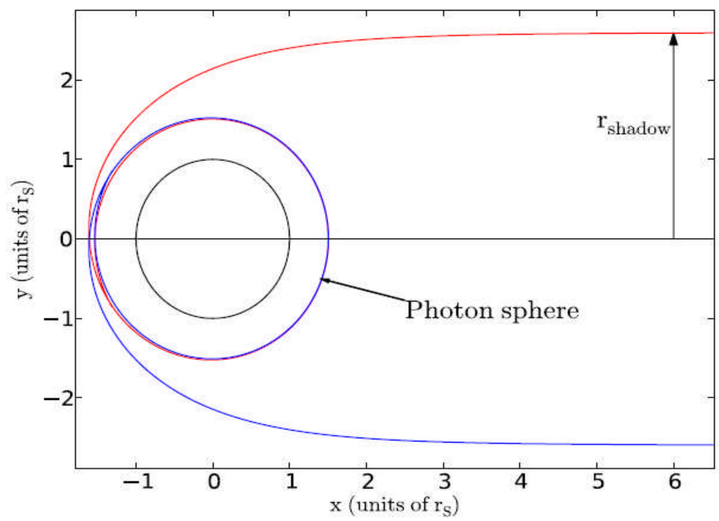


Figure 1. Fig. 1 of T. Lacroix & J. Silk[4]. Shadow of a black hole. The radius of the shadow is the minimum impact parameter of a light ray escaping the black hole, so the shadow is a disk representing the black hole as seen by the observer. The circular orbit lies on the so-called photon sphere. The black circle represents the horizon.

Overlay ofthe NDS projections on the Lacroix & Silk graph.

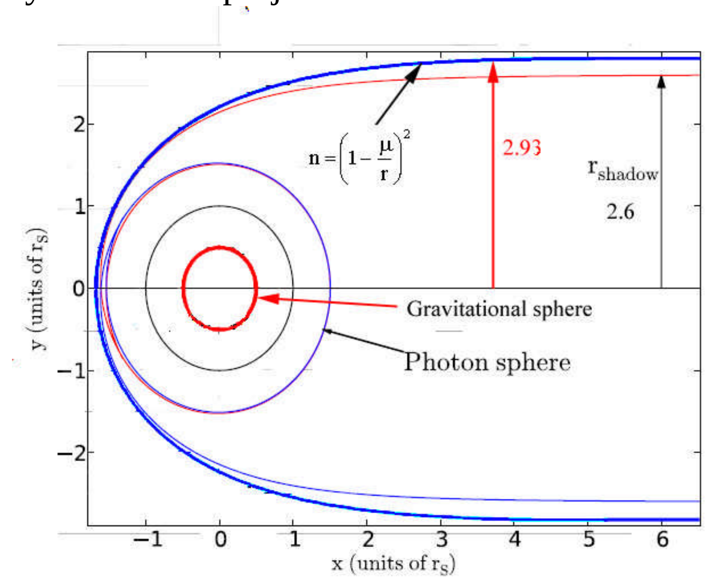


Figure 2. *The NDS photon shadow radius of (2.93 r_S) is slightly larger than its BH equivalent (2.6 r_S).*

Results

Photons leaving the surface of an NDS **vertically**, **escape into space**, however if photon leave the source at a **slight angle**, gravitation can bend the trajectory into **an orbit** at a level below the maximum photon orbit. For a neutron star the size of Sag A*, the **maximum angle from vertical for a photon to escape is about 0.004 rad.** At that angle the photon will go into the maximum photon orbit, but if the angle is greater, the orbit lies between the surface and the maximum orbit, as shown in This figure. Although there may be a stability issue, as the photons trajectory curves perpendicular to the radius vector, the structure of Snell's law will not allow the radial velocity to become negative, therefore the trajectory at any elevation below the maximum photon orbit is a circular orbit.

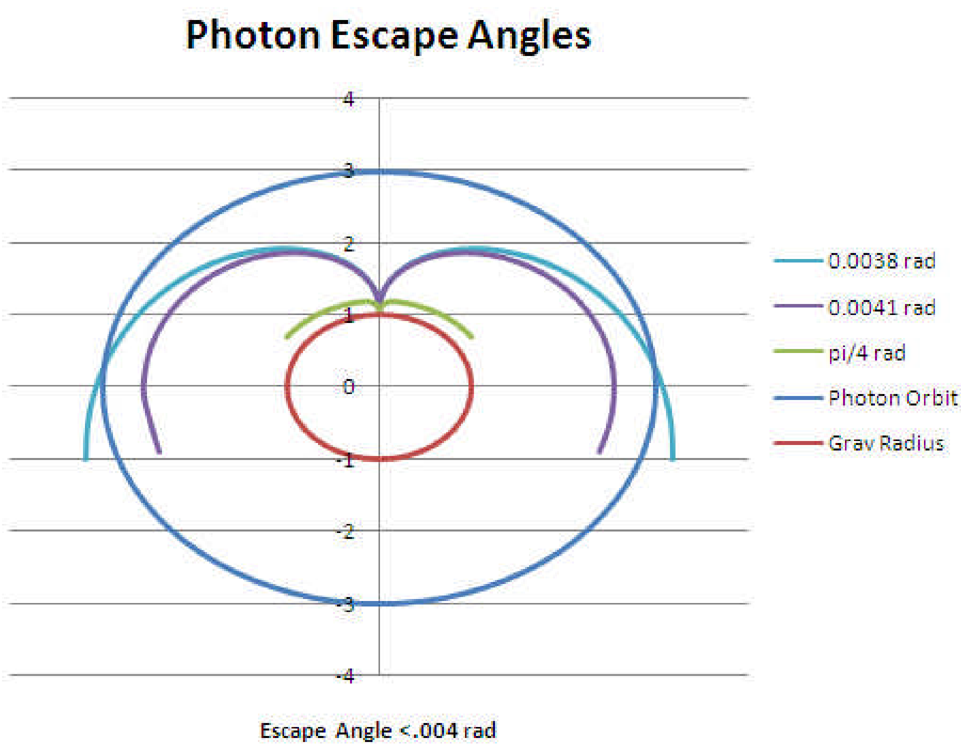


Figure 3. *Illustration of the photon emission angle for photons that escape, and those photons that would be captured in orbit.*

Photon Trajectories
Escape Angle 0 - 0.004 Rad

It is noted that in the absence of intervening or accretion material, GR predicts that there will be a photon void emanating inside the shadow for a BH, thus a dark image radius of about $2.6\,r_S$. This is because there are no photons originating from the sphere of the black hole, and any interloping photon would be captured.

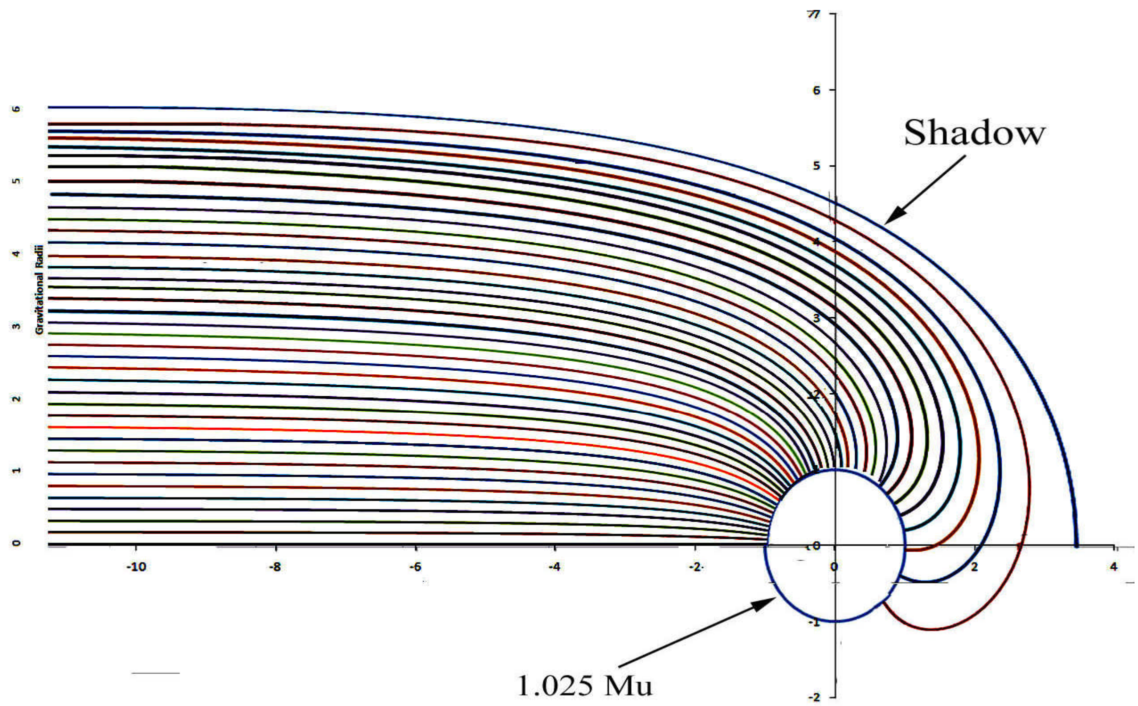


Figure 4. ***shows the ray trace trajectory of photons leaving the surface with an angle to the vertical between 0.0 and 0.004 rad, and crossing perpendicular to a distant plane.***

Calculating the Relative Luminosity

L is the apparent external luminosity, L_S is the luminosity at the surface , If $\Delta\theta_S$ is the angle separating the photons at the surface, and $\Delta\theta_P$ is the angle between the same photons when they escape the system at a distant escape plane. Then $L \sim (\Delta\theta_P / \Delta\theta_S)^2$

This figure, illustrates the angular dispersion difference between photons leaving vertically and those from lower angles wrapping around the star.

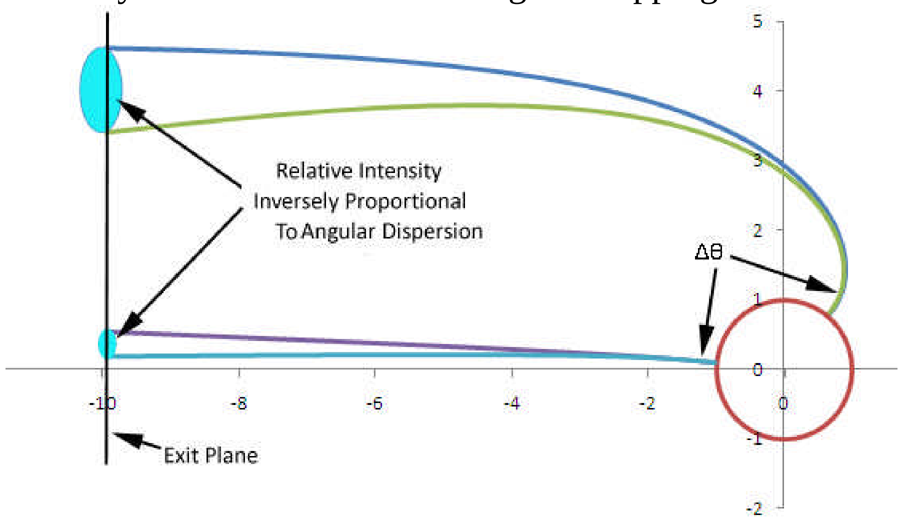
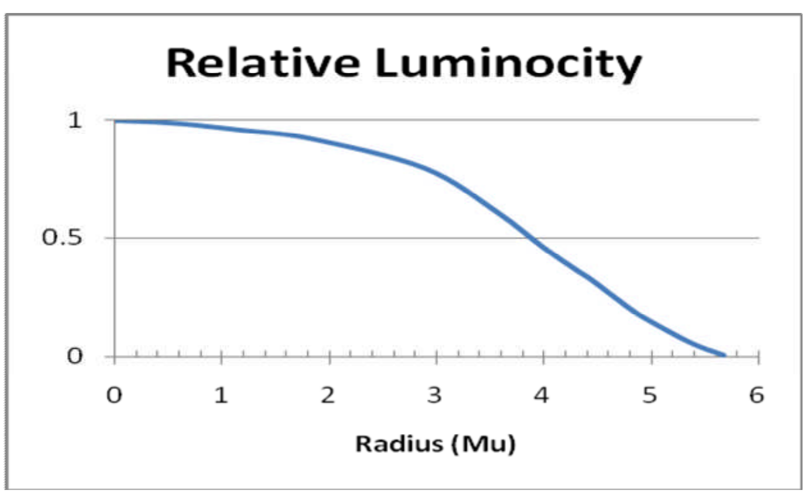


Figure 5. *Illustrating the luminosity at the escape plane as a function of the spherical exit position. (Exaggerated)*



This is the plot of the intensity profile of photons exiting perpendicular to a distant plane as measured from the center of the image

Overlay Intensity With Escape Angles

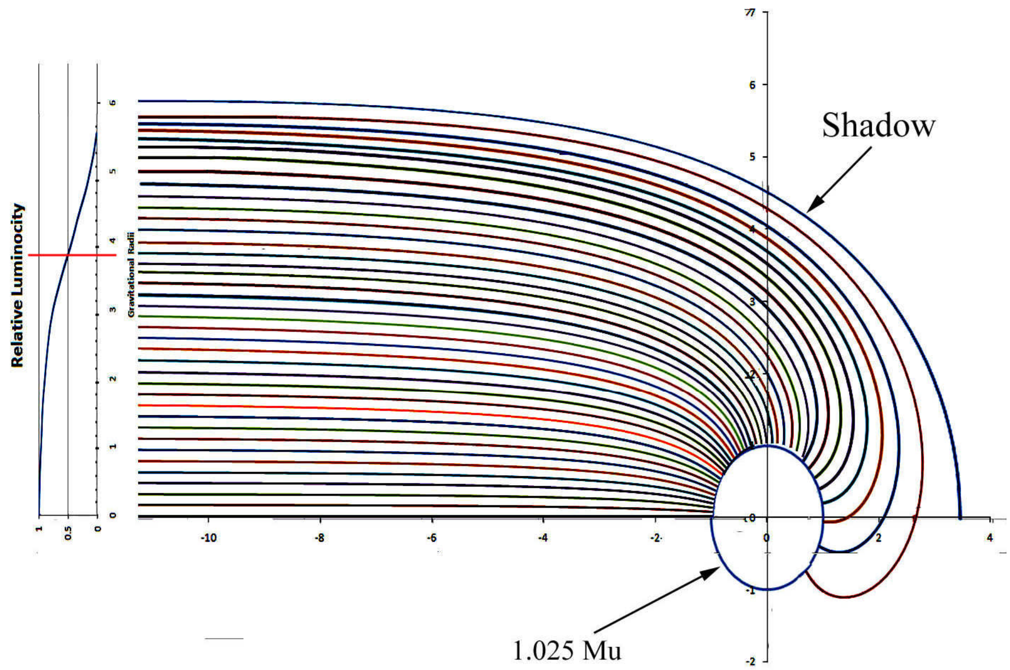


Figure 7. *This illustrates the exit position on the exit plane of a photon trajectory originating at positions on the surface. The relative intensity of points on the surface as viewed at a large distance perpendicular to that plane is plotted on the left side of the trajectory. The red line is the half maximum intensity radius in* μ.

Comparisons with Current Measurements

Figure 8 is the graph from Doeleman et al [9], illustrating the apparent diameter vs. Black Hole diameter. The red line represents the observed size (FWHM) using the data from 2007 and 2009 with three of the EHT telescopes in [ARO/SMT, CARMA, JCMT]. The intensity vs. radius for the NDS from is plotted to the right of the graphs showing the Full Width Half Maximum diameter to be about the same as the current low resolution measurements.

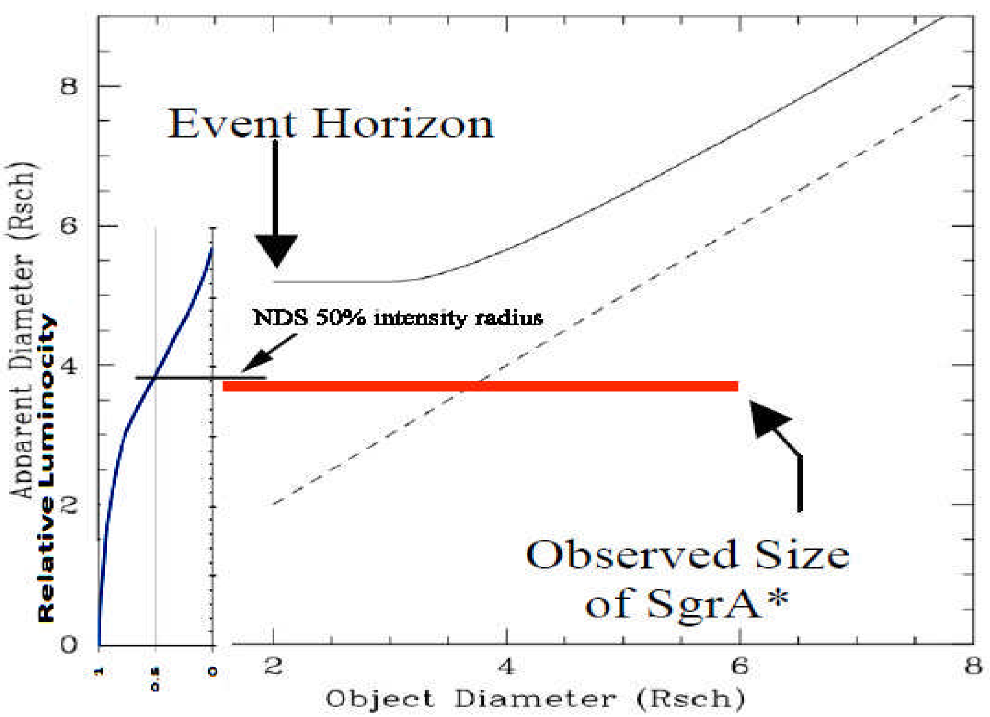


Figure 8. *(* Fig. 1 of Doeleman et al [9]*) A symmetric emitting surface surrounding a black hole is gravitationally lensed to appear larger than its true diameter. Here the apparent size is plotted as a function of the actual object size. The solid black line shows the apparent diameter with lensing by a non-spinning black hole, and the dashed line with no lensing effects included. The intrinsic size of Sgr A* observed with1.3mm VLBI. (horizontal red line), is smaller than the minimum apparent size of the black hole event horizon (labeled 'Event Horizon')* [ARO/SMT, CARMA, JCMT

EHT Simulated Image

Dimitrios Psalti , et al, [11].

The explanation by the EHT team for the small size of the currently observed low resolution image is that the image is of the edge on view on the accretion disk, and not the rim of the Black Hole.

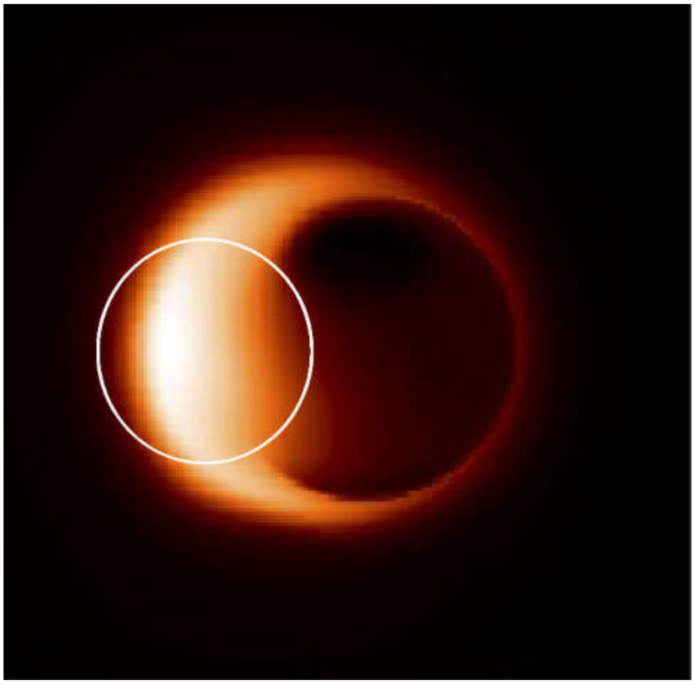


Figure 9. *Black Hole: EHT simulated image of the accretion generated radiation of Sag A**[11]. *The black Hole shadow is black area on the right with the accretion generated radiation to the left. The white 3.7* r_S *diameter circle (added by this author) is the EHT team's proposed source of the current measurements*

NDS Simulated Image

Neutron Dark Star

The expected image of the NDS star (figure 10), should be centered at the center of mass, and spherically symmetric, having a Full Width Half Maximum diameter of about 3.85 r_S and a shadow diameter of about 5.86 r_S

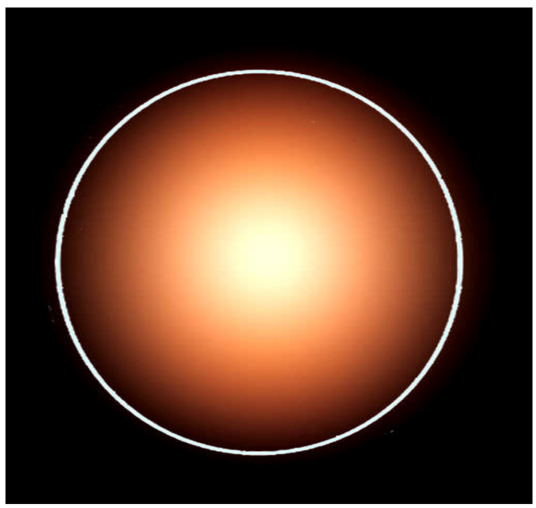
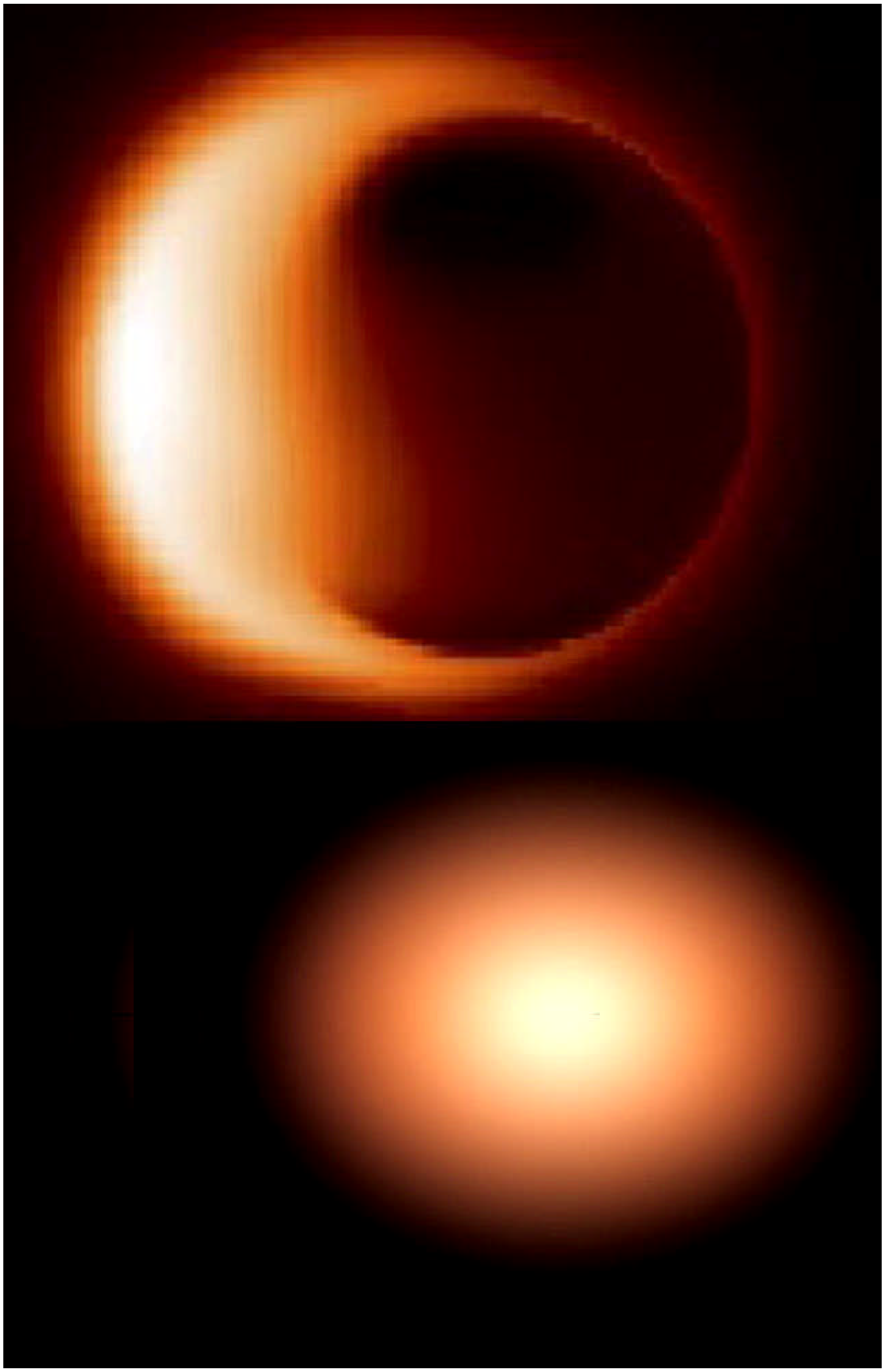


Figure 10. *NDS: Simulated image of the escaping radiation from the Neutron Dark Star. The Full Width Half Maximum diameter is about 3.85 Schwarzschild radii. The white circle is the NDS* shadow diameter of about 5.86 r_S

Image Comparison
Black Hole Vs Neutron Dark Star



Top:: EHT Team expected Black Hole accretion image.
Bottom : NDS Neutron Dark Star image.